Ghost-free superconformal action for multiple $M2$-branes

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ABSTRACT: The Bagger-Lambert construction of $\mathcal{N} = 8$ superconformal field theories (SCFT) in three dimensions is based on 3-algebras. Three groups of researchers recently realized that an arbitrary semisimple Lie algebra can be incorporated by using a suitable Lorentzian signature 3-algebra. The SU(N) case is a candidate for the SCFT describing coincident M2-branes. However, these theories contain ghost degrees of freedom, which is unsatisfactory. We modify them by gauging certain global symmetries. This eliminates the ghosts from these theories while preserving all of their desirable properties. The resulting theories turn out to be precisely equivalent to $\mathcal{N} = 8$ super Yang-Mills theories.

KEYWORDS: Conformal Field Models in String Theory, Chern-Simons Theories.

Contents

1. Introduction

Bagger and Lambert $[1-3]$, as well as Gustavsson $[4, 5]$ discovered the general rules for constructing an action for a three-dimensional theory with $OSp(8|4)$ superconformal symmetry. Their solution is based on a 3-algebra, which is characterized by structure constants $f^{ABC}{}_D$ and a metric h_{AB}. The initial assumption was that the metric should be positive definite. This led to the discovery of a theory with SO(4) gauge symmetry $[2]$. Its full superconformal symmetry was verified in $[6]$, which also conjectured its uniqueness. The uniqueness of this theory was proved in $[7, 8]$. A proposal for its physical interpretation in terms of M2-branes in M-theory at an M-fold singularity has been given in $[9, 10]$.

These developments left unresolved the question whether it is possible to give a Lagrangian description of the conformal field theory associated with coincident M2-branes in flat 11-dimensional spacetime. That theory is known to correspond to the IR fixed point of $\mathcal{N} = 8$ super Yang-Mills theory. The question is whether there is a dual formulation of this fixed-point theory. The only apparent way of evading the uniqueness theorem is to consider 3-algebras with an indefinite signature metric. This possibility was examined by three different groups $[11-13]$, who proposed a new class of theories based on a 3-algebra with Lorentzian signature. The generators of the 3-algebra are the generators of an arbitrary semisimple Lie algebra plus two additional null generators $T^{\pm}$. The theory based on the 3-algebra associated to the gauge group SU(N) or $\Upsilon(N)$ looks like a good candidate for the theory of N coincident M2-branes, except for the fact that it contains unwanted negative norm states in the physical spectrum. This makes the theory nonunitary even though these states do not contribute to loops. Subsequent papers discussing the interpretation and application of Lorentzian 3-algebras include $[14]$–$[25]$. In particular, $[25]$ proved that the Lorentzian 3-algebras considered in $[11-13]$ are the *only* indecomposable Lorentzian 3-algebras (aside from the obvious SO(3, 1) variant of the Bagger-Lambert theory).

In this paper we propose modifying the construction in [11 – 13] by gauging certain global symmetries.[1] We claim that this eliminates the unwanted ghost degrees of freedom while preserving all of the other symmetries. In section 2 we explain the basic idea of our construction in a simplified model. Section 3 applies the same procedure to the theory of interest.

2. The basic idea

After integrating out certain auxiliary fields, the theory proposed in [11 – 13] contains terms of the form

$$S \sim \int d^3x \left(-\phi_+^{-2} \mathrm{Tr}(F^2) + \partial^\mu \phi_+ \partial_\mu \phi_- \right)$$

This has manifest scale invariance if $\phi_\pm$ have dimension $1/2$. This theory has a ghost degree of freedom, which (ignoring the first term) is reminiscent of the one contained in the covariant gauge-fixed string world-sheet theory prior to imposing the Virasoro constraints. In the present case, there are no Virasoro constraints, so the theory needs to be modified if we wish to make sense of it.

An important clue is that this theory has a global symmetry given by a constant shift of the field ϕ_-. Our proposal is to modify this theory by gauging this symmetry through the inclusion of a dimension $3/2$ Stückelberg field C_μ

$$S \sim \int d^3x \left(-\phi_+^{-2} \mathrm{Tr}(F^2) + \partial^\mu \phi_+ (\partial_\mu \phi_- - C_\mu) \right).$$

The gauge symmetry is simply given by

$$\delta \phi_- = \Lambda \qquad \delta C_\mu = \partial_\mu \Lambda.$$

Classically, this theory is conformally invariant. (In the case of the M2-brane theory in the next section the conformal symmetry is expected to survive in the quantum theory.) This theory can be gauge fixed by setting $\phi_- = 0$. Integrating out C_μ gives a delta functional imposing the constraint $\partial_\mu \phi_+ = 0$. Thus, ϕ_+ is a constant, which is determined by a boundary condition. Calling the constant g_{YM}, we are left with pure Yang-Mills theory

$$S \sim -g_{\mathrm{YM}}^{-2} \int d^3x \mathrm{Tr}(F^2).$$

The Yang-Mills theory is not conformally invariant, of course, since g_{YM} is dimensionful. However, this construction shows that it arises from spontaneous breaking of the conformal symmetry.

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[1] After this work had been completed, Hirosi Ooguri informed us that Masahito Yamazaki is also considering this possibility.

3. Modifying the BL theory

Using the notation of [12], we start with the following Bagger-Lambert theory based on a family of 3-algebras with Lorentzian metric:

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2}\mathrm{Tr}\left(D_\mu X^I D^\mu X^I\right) + D_\mu X_+^I D^\mu X_-^I + \frac{i}{2}\mathrm{Tr}\left(\bar\Psi\Gamma^\mu D_\mu\Psi\right) \\
& -\frac{i}{2}\bar\Psi_+\Gamma^\mu D_\mu\Psi_- - \frac{i}{2}\bar\Psi_-\Gamma^\mu D_\mu\Psi_+ + \epsilon^{\mu\nu\lambda}\mathrm{Tr}\left(\mathcal{B}_\lambda\left(\partial_\mu\mathcal{A}_\nu - [\mathcal{A}_\mu,\mathcal{A}_\nu]\right)\right) \\
& -\frac{1}{12}\mathrm{Tr}\left(X_+^I\left[X^J,X^K\right] + X_+^J\left[X^K,X^I\right] + X_+^K\left[X^I,X^J\right]\right)^2 \\
& +\frac{i}{2}\mathrm{Tr}\left(\bar\Psi\Gamma_{IJ}X_+^I\left[X^J,\Psi\right]\right) + \frac{i}{4}\mathrm{Tr}\left(\bar\Psi\Gamma_{IJ}\left[X^I,X^J\right]\Psi_+\right) - \frac{i}{4}\mathrm{Tr}\left(\bar\Psi_+\Gamma_{IJ}\left[X^I,X^J\right]\Psi\right),
\end{aligned}
\tag{3.1}
$$

where $I = 1,\ldots,8$ are the transverse coordinates and $X_\pm^I = \frac{1}{\sqrt{2}}\left(X_0^I \pm X_1^I\right)$. The covariant derivatives are defined as

$$
D_\mu X^I = \partial_\mu X^I - 2\left[\mathcal{A}_\mu, X^I\right] - \mathcal{B}_\mu X_+^I,
\tag{3.2a}
$$

$$
D_\mu X_-^I = \partial_\mu X_-^I - \mathrm{Tr}\left(\mathcal{B}_\mu X^I\right),
\tag{3.2b}
$$

$$
D_\mu X_+^I = \partial_\mu X_+^I
\tag{3.2c}
$$

and similarly for the fermions. Note that this theory has a noncompact gauge group whose Lie algebra is a semidirect sum of any ordinary Lie algebra $\mathfrak{g}$ of a compact Lie group $\mathcal{G}$, and $\dim(\mathfrak{g})$ abelian generators. The gauge field $\mathcal{A}_\mu$ is associated with the compact part, while the gauge field $\mathcal{B}_\mu$ is associated with the noncompact part. This theory was recently proposed in [11–13]. Various details of this Lagrangian, including its field content, gauge symmetry, and supersymmetry transformations, are given in the appendix. Like all BL theories, it has $\mathcal{N} = 8$ supersymmetry, scale invariance, conformal invariance, and SO(8) R-symmetry. These combine to give the supergroup $OSp(8|4)$. The theory also has parity invariance. At the same time, it does not admit any tunable coupling constant, since any coupling constant can be absorbed in field redefinitions. Furthermore $\mathcal{G}$ can be chosen to be any compact Lie group. These are special features that are not shared by the SO(4) BL theory, which is based on a 3-algebra with a positive-definite metric.

Despite the numerous properties which make this theory a promising candidate for describing multiple M2-branes in flat space, it has one very troubling feature. To see this, consider the fields X_-^I and Ψ_-. Note that the full dependence on these fields is given by:

$$
\mathcal{L}_- = -i\bar\Psi_+\Gamma^\mu\partial_\mu\Psi_- + \partial^\mu X_+^I\partial_\mu X_-^I.
\tag{3.3}
$$

As it stands, these terms describe propagating ghost degrees of freedom, which makes the theory unsatisfactory, since it is not unitary. At this point, it is useful to observe that the action has the following global shift symmetries (pointed out in [12]):

$$
\delta X_-^I = \Lambda^I \quad\text{and}\quad \delta\Psi_- = \eta.
$$

Also note that Ψ_- and X_-^I do not appear in any of the gauge or SUSY transformations of the other fields. We will show that it is possible to eliminate the ghosts from the theory,

while preserving all of its desirable properties, by promoting these global shift symmetries to local symmetries.

To gauge the global shift symmetries described above we introduce two new gauge fields: a vector field C_μ^I in the vector representation of SO(8), and a 32-component Majorana-Weyl spinor χ satisfying $\Gamma^{012}\chi = -\chi$. These appear in two new terms which we add to the Lagrangian:

$$\mathcal{L}_{\text{new}} = \bar{\Psi}_+ \chi - \partial^\mu X_+^I C_\mu^I. \tag{3.4}$$

Note that C_μ^I must have dimension 3/2 and χ must have dimension 2 to preserve scale invariance. The new local shift symmetries are

$$\delta X_-^I = \Lambda^I, \quad \delta C_\mu^I = \partial_\mu \Lambda^I \tag{3.5}$$

and

$$\delta \Psi_- = \eta, \quad \delta \chi = i\Gamma^\mu \partial_\mu \eta. \tag{3.6}$$

There is one additional local symmetry of eq. (3.4), which is relatively trivial, namely

$$\delta C_\mu^I = \partial^\rho \tilde{\Lambda}_{\mu\rho}^I, \quad \text{where} \quad \tilde{\Lambda}_{\mu\rho}^I = -\tilde{\Lambda}_{\rho\mu}^I. \tag{3.7}$$

C_μ^I and χ are invariant under the original gauge symmetries.

Now let us consider the supersymmetry of the modified theory. The supersymmetry transformations of all the old fields are unchanged. In particular,

$$\delta X_+^I = i\bar{\varepsilon}\Gamma^I \Psi_+ \tag{3.8}$$

and

$$\delta \Psi_+ = \Gamma^\mu \partial_\mu X_+^I \Gamma^I \varepsilon. \tag{3.9}$$

The supersymmetries of the new gauge fields must be defined in such a way that $\mathcal{L}_{\text{new}}$ is invariant. We will find that the resulting supersymmetry algebra closes on shell when one takes account of the new gauge symmetries. Under supersymmetry

$$\delta C_\mu^I = \bar{\varepsilon}\Gamma^I \Gamma_\mu \chi \tag{3.10}$$

and

$$\delta \chi = i\Gamma^I \varepsilon\, \partial^\mu C_\mu^I. \tag{3.11}$$

Using these four transformation rules, it is easy to see that both $\mathcal{L}_{\text{new}}$ and the equations of motion are supersymmetric.

We will now check the closure of all the algebras. The fact that the supersymmetry variations of C_μ^I and χ are not invariant under the new gauge transformations implies that the supersymmetry transformations do not commute with these gauge transformations. Specifically, one finds that

$$[\delta(\Lambda), \delta(\varepsilon)] = \delta(\eta), \quad \text{where} \quad \eta = \Gamma^\mu \Gamma^I \partial_\mu \Lambda^I \varepsilon \tag{3.12}$$

and

$$[\delta(\eta), \delta(\varepsilon)] = \delta(\Lambda) + \delta(\tilde{\Lambda}) \quad \text{where} \quad \Lambda^I = i\bar{\varepsilon}\Gamma^I \eta \quad \text{and} \quad \tilde{\Lambda}_{\mu\rho}^I = i\bar{\varepsilon}\Gamma^I \Gamma_{\mu\rho} \eta. \tag{3.13}$$

The supersymmetry algebra is slightly affected, as well. Specifically, we find that

$$[\delta(\varepsilon_1), \delta(\varepsilon_2)]C_\mu^I = \delta(\xi)C_\mu^I + \delta(\tilde{\Lambda})C_\mu^I, \tag{3.14}$$

where $\xi^\rho = 2i\bar{\varepsilon}_1\Gamma^\rho\varepsilon_2$, as usual, and $\tilde{\Lambda}_{\mu\rho}^I = \xi_\mu C_\rho^I - \xi_\rho C_\mu^I$. Similarly, for χ we find that

$$[\delta(\varepsilon_1), \delta(\varepsilon_2)]\chi = \delta(\xi)\chi + \delta(\eta)\chi, \tag{3.15}$$

where $\eta = \left(-\bar{\epsilon}_1\Gamma^\mu\epsilon_2\Gamma_\mu + \frac{1}{4}\bar{\epsilon}_1\Gamma^{LM}\epsilon_2\Gamma_{LM}\right)\chi$. One also finds that requiring the on-shell closure of the commutator $[\delta(\varepsilon_1), \delta(\varepsilon_2)]\Psi_-$ gives the expected equation of motion for Ψ_- after noting that the commutator receives a contribution from $\delta(\eta)\Psi_-$. In summary, we have verified that the supersymmetries close on shell into translations, the old gauge transformations, and the new gauge transformations given by Eqs (3.5)–(3.7).

4. Discussion

After modifying the theory by introducing the new gauge fields C_μ and χ, it still has scale invariance, $\mathcal{N} = 8$ supersymmetry, no coupling constant, and can accommodate any Lie group in its gauge group, which are all desirable properties for describing multiple M2-branes in flat space. In addition, we can use the new gauge symmetries to make the gauge choices

$$X_-^I = \Psi_- = 0.$$

This removes the kinetic terms for the ghosts and changes the supersymmetry transformations for C_μ and χ by induced gauge transformations, i.e. $\delta C_\mu^I = \bar{\epsilon}\Gamma^I\Gamma_\mu\chi + \partial_\mu\Lambda^I$ and $\delta\chi = i\Gamma^I\epsilon\partial^\mu C_\mu^I + i\Gamma^\mu\partial_\mu\eta$ for appropriate choices of Λ^I and η. Furthermore, the equations of motion that come from varying the new fields are

$$\partial_\mu X_+^I = 0, \quad \Psi_+ = 0.$$

The first equation implies that the X_+^I is a constant. Any nonzero choice spontaneously breaks conformal symmetry and breaks the R-symmetry to an unbroken SO(7) subgroup. On the other hand, the choice $X_+^I = 0$ gives a free theory.

We can use the SO(8) R-symmetry to choose the nonzero component of X_+^I to be in the 8 direction, $X_+^I = v\delta^{I8}$. Also, the noncompact gauge fields, $\mathcal{B}$, which appear quadratically can be integrated out. This leaves a maximally supersymmetric 3d Yang-Mills theory with SO(7) R-symmetry:

$$\mathcal{L} = -\frac{1}{4v^2}\text{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{1}{2}\text{Tr}\left(D'_\mu X^i D'_\mu X^i\right) + \frac{i}{2}\text{Tr}\left(\bar{\Psi}\Gamma^\mu D'_\mu\Psi\right)$$
$$+\frac{i}{2}\text{Tr}\left(\bar{\Psi}\Gamma_{8i}\left[X^i, \Psi\right]\right) - \frac{v^2}{4}\text{Tr}\left(\left[X^i, X^j\right]\right)^2 \tag{4.1}$$

where the index $i = 1, \ldots, 7$, and D'_μ and $F_{\mu\nu}$ depend only the massless gauge field $\mathcal{A}$ associated with the maximally compact subgroup of the original gauge group. Note that this is an exact result — not just the leading term in a large-v expansion. This is a supersymmetric generalization of the toy model described in section 2.

To summarize, in this paper we have proposed a modification of the Bagger-Lambert theory that removes the ghosts when the 3-algebra has a Lorentzian signature metric, thus ensuring unitarity. Such theories evade the no-go theorem, which states that there is essentially only one nontrivial 3-algebra with positive-definite metric. Our modification of the Lorentzian 3-algebra theories in $[11-13]$ breaks the conformal symmetry spontaneously and reduces them to maximally supersymmetric 3d Yang-Mills theories.[2] This result is somewhat disappointing inasmuch as it means that we are no closer to the original goal of understanding the $v \to \infty$ IR fixed-point theory that describes coincident M2-branes in 11 noncompact dimensions. As things stand, it appears that the BL SO(4) theory is the only genuinely new maximally supersymmetric superconformal theory. Of course, one should still explore whether there are other 3-algebras (whose metric is neither positive-definite not Lorentzian) that open new possibilities.

Note added: After this paper was first posted, two related papers appeared [26, 27]. Also, a paper by Aharony et al. appeared that introduces a very promising class of theories with $\mathcal{N} = 6$ superconformal symmetry [28]. It proposes that these theories actually have $\mathcal{N} = 8$ superconformal symmetry (implemented in a very subtle manner) in the appropriate cases.

Acknowledgments

A. BL theory for general Lie algebras

In this appendix, we follow the notation of [12]. The Lagrangian of a BL-theory is completely specified once a 3-algebra with a metric is given. The structure constants of the 3-algebra $f^{ABC}{}_D$ must satisfy the fundamental identity and $f^{ABCE} = f^{ABC}{}_D h^{DE}$, where h^{DE} is the 3-algebra metric, must be totally antisymmetric. In [12], the 3-algebra is constructed from an ordinary Lie algebra $\mathfrak{g}$ by adding two generators to $\mathfrak{g}$ called T^+ and T^- so that the 3-algebra has dimension $dim(\mathfrak{g}) + 2$. Its structure constants are given in terms of the $\mathfrak{g}$-structure constants $f^{ab}{}_c$ as

$$f^{+ab}{}_c = f^{ab}{}_c, \tag{A.1}$$

with all other nonzero components of $f^{ABC}{}_D$ related by permuting, raising, or lowering indices. The generators of $\mathfrak{g}$ satisfy

$$\left[T^a, T^b\right] = f^{ab}{}_c T^c, \tag{A.2}$$
$$\mathrm{Tr}\left(T^a T^b\right) = \delta^{ab}.$$

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[2]Reference [13] observed that if one chooses X_+^I to be constant and Ψ_+ to be zero, then the theory reduces to N=8 SYM. However, they did not deduce these choices from an action principle.

Field	3d World Volume	SO(8)	$\mathfrak{g}$	Dimension
$X_{\pm}^I$	Scalar	8_{v}	Singlet	1/2
X^I	Scalar	8_{v}	Adjoint	1/2
$\Psi_{\pm}$	Spinor	8_{s}	Singlet	1
Ψ	Spinor	8_{s}	Adjoint	1
$\mathcal{A}_{\mu}$	Gauge field	1	Adjoint	1
$\mathcal{B}_{\mu}$	Gauge field	1	Adjoint	1

Table 1: Field content of the theory.

The invariant metric of the 3-algebra is given by

$$h^{+-} = -1, \qquad h^{++} = 0, \qquad h^{--} = 0, \qquad h^{ab} = \delta^{ab}. \tag{A.3}$$

With the choice of structure constants and 3-algebra metric given above, the BL theory reduces to the Lagrangian given in eq. 1. The field content of the theory is summarized in table 1.

The gauge transformations are

$$\delta X^I = 2\left[\Lambda, X^I\right] + MX_+^I, \tag{A.4a}$$

$$\delta X_-^I = \mathrm{Tr}\left(MX^I\right), \tag{A.4b}$$

$$\delta X_+^I = 0, \tag{A.4c}$$

$$\delta \Psi = 2\left[\Lambda, \Psi\right] + M\Psi_+, \tag{A.4d}$$

$$\delta \Psi_- = \mathrm{Tr}\left(M\Psi\right), \tag{A.4e}$$

$$\delta \Psi_+ = 0. \tag{A.4f}$$

$$\delta \mathcal{A}_\mu = \partial_\mu \Lambda + 2\left[\Lambda, \mathcal{A}_\mu\right], \tag{A.4g}$$

$$\delta \mathcal{B}_\mu = \partial_\mu M + 2\left[M, \mathcal{A}_\mu\right] + 2\left[\Lambda, \mathcal{B}_\mu\right], \tag{A.4h}$$

where Λ and M are infinitesimal matrices in the adjoint of $\mathfrak{g}$. The matrix Λ generates the $\mathcal{G}$ gauge transformations while M generates the noncompact subgroup transformations.

Finally, the $\mathcal{N} = 8$ SUSY transformations (consistent with scale invariance) are

$$\delta \mathcal{A}_\mu = \frac{i}{2}\bar{\varepsilon}\Gamma_\mu\Gamma_I\left(X_+^I\Psi - X^I\Psi_+\right), \tag{A.5a}$$

$$\delta \mathcal{B}_\mu = i\bar{\varepsilon}\Gamma_\mu\Gamma_I\left[X^I, \Psi\right], \tag{A.5b}$$

$$\delta X_{\pm}^I = i\bar{\varepsilon}\Gamma^I\Psi_{\pm}, \tag{A.5c}$$

$$\delta X^I = i\bar{\varepsilon}\Gamma^I\Psi, \tag{A.5d}$$

$$\delta \Psi_+ = \partial_\mu X_+^I\Gamma^\mu\Gamma^I\varepsilon, \tag{A.5e}$$

$$\delta \Psi_- = D_\mu X_-^I\Gamma^\mu\Gamma^I\varepsilon - \frac{1}{3}\mathrm{Tr}\left(X^IX^JX^K\right)\Gamma_{IJK}\epsilon, \tag{A.5f}$$

$$\delta \Psi = D_\mu X^I\Gamma^\mu\Gamma^I\varepsilon - \frac{1}{2}X_+^I\left[X^J, X^K\right]\Gamma_{IJK}\epsilon. \tag{A.5g}$$

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